Risk aversion of market makers and asymmetric information

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There are three types of agents on the market:

- Noisy/liquidity traders: the noise demand is given by $Z_t = \sigma B_t$.
- Informed investor: observes $\mathcal{F}^I_t = \mathcal{F}^S_t \vee \sigma(V)$ and is risk-neutral, i.e. she solves

$$\sup_{X \in \mathcal{A}(H)} \mathbb{E}^v \left[W_1^X \right] = \sup_{X \in \mathcal{A}(H)} \mathbb{E}^v \left[(V - S_1)X_1 + \int_0^1 X_s dS_s \right],$$

where $\mathbb{E}^v$ is the expectation using the probability measure of the insider who is given the realisation $V = v$.

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  - Observe $\mathcal{F}_t^M = \mathcal{F}_t^Y$ where $Y = \sigma B + X$ is the total demand process.
  - Set the price, $S_t = H(t, Y_t)$ according to zero utility gain condition, i.e.

$$\mathbb{E}[U(G_t) - U(G_s)|\mathcal{F}_s^M] = 0, \ \forall s \leq t,$$

where

$$\begin{aligned} G_t : &= -\frac{1}{N}\int_0^t Y_s \, dH(s, Y_s) - \mathbf{1}_{[t=1]}\frac{Y_1}{N}(V - H(Y_1, 1)), \\ \mathbb{P}[A] &= \int \mathbb{P}^v[A] d\Phi(f^{-1}(v)), \end{aligned}$$

with Φ being a cdf of a standard normal.

Definition of Equilibrium

Definition

A couple (H^*, X^*) is an equilibrium if H^* is an increasing function, X^* is an absolutely continuous process (+some technical conditions), and the following conditions are satisfied:

1. *Market efficiency condition:* given X^*, H^* satisfies zero-utility gain condition.
2. *Insider optimality condition:* given H^*, X^* solves the insider optimization problem:

$$\mathbb{E}^v[W_1^{X^*}] = \sup_{X \in \mathcal{A}(H^*)} \mathbb{E}^v[W_1^X].$$

Theorem

Suppose f is either bounded with a continuous derivative or linear. Then there exists a function H such that

$$H_t + \frac{1}{2}\sigma^2 H_{yy} = 0, \text{ and } H(1,\xi_1) \stackrel{d}{=} f(N(0,1)) \qquad (1)$$

where ξ is a strong solution to (β is a Brownian Motion)

$$\xi_t = \sigma\beta_t - \int_0^t \frac{\sigma^2\rho}{2N}\xi_t H_y(s,\xi_s)\, ds. \qquad (2)$$

Moreover, this H and

$$\begin{aligned} Y_t &= \sigma B_t - \int_0^t \left[\frac{\sigma^2\rho}{2N} Y_s H_y(s, Y_s) + \frac{p_y}{p}(s, Y_s; 1, H^{-1}(1, V))\right] ds \\ &=: \sigma B_t + X_t^*, \end{aligned} \qquad (3)$$

where $p(s, y; t, z) = \mathbb{P}[\xi_t \in dz | \xi_s = y]$, constitute an equilibrium.

On zero-utility gain

Recall that

- the zero expected utility gain for a market maker means that $U(G)$ is a $(\mathcal{F}^M, \mathbb{P})$-martingale, where

$$G_t := -\frac{1}{N}\int_0^t Y_s\, dH(s, Y_s) - \mathbf{1}_{[t=1]}\frac{Y_1}{N}(V - H(1, Y_1)).$$

- In filtration $\mathcal{F}^M$ and under measure $\mathbb{P}$ total demand is

$$dY_t = \sigma dB_t^Y - \frac{\sigma^2\rho}{2N} Y_t H_y(t, Y_t)\, dt. \tag{4}$$

- Direct application of Ito yields

$$dU(G_t) = -\sigma U'(G_t)\frac{Y_t}{N} H_y(t, Y_t) dB_t^Y$$

First implications of equilibrium

- Net demand flow has a drift in its own filtration:

$$dY_t^* = \sigma dB_t^Y - \frac{\sigma^2 \rho}{2N} Y_t^* H_y^*(t, Y_t^*)\, dt,$$

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- There are, in general, systematic changes in the market depth.
- In the limit $\frac{\rho}{N} \to 0$ Kyle's result is retrieved.

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- Thus, to have $H(1, \xi_1) \overset{d}{=} f(\mathcal{N}(0, 1))$, λ must solve

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- And the equilibrium demand Y^* solves

$$dY_t^* = \sigma dB_t + \frac{\rho\sigma^2}{2N} \frac{a\eta - \lambda Y_t^* \cosh\left(\frac{\rho\sigma^2\lambda}{2N}(1-t)\right)}{\sinh\left(\frac{\rho\sigma^2\lambda}{2N}(1-t)\right)}. \tag{6}$$

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- The resilience is the speed with which the prices converge to the fundamental value. In Kyle's equilibrium it equals

$$\frac{1}{1-t} \left(= -\frac{\frac{d}{dt}(V - \mathbb{E}^v[Y_t^*])}{V - \mathbb{E}^v[Y_t^*]} \right)$$

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- The profit of the insider is σ.

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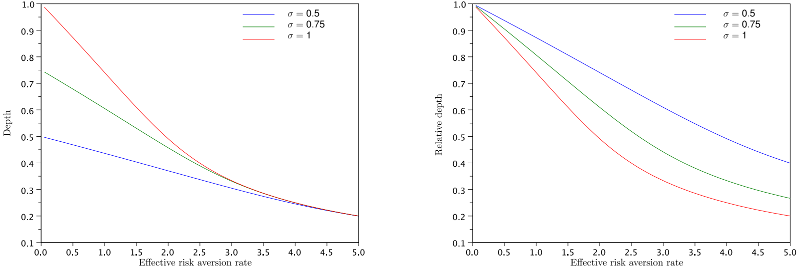
- The market depth is $\frac{4N}{\rho}$.

Market depth

The depth of the market

is still constant and equal to $\frac{1}{\lambda}$, where λ solves $1 - e^{-\frac{\rho\sigma^2}{N}\lambda} = \frac{\rho}{N}\frac{1}{\lambda}$.

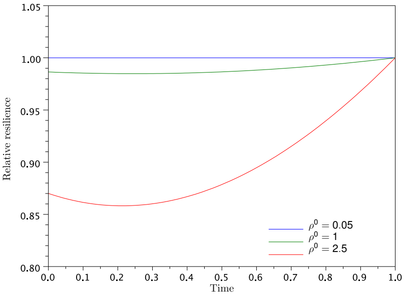


Market becomes less liquid as the market makers become more risk averse. The panel on the right depicts how this loss of liquidity, measured in terms of the relative depth, i.e. $\frac{1}{\lambda\sigma}$, depends on the risk aversion and noise volatility.

Resilience

Resilience is given by

$$\frac{d}{dt}\log\left(\eta - \mathbb{E}^{v}[\lambda Y_t^*]\right) = \frac{\lambda^0 \cosh(\lambda^0 t)}{\sinh(\lambda^0) - \sinh(\lambda^0 t)}, \ \lambda^0 := \frac{\rho\sigma^2\lambda}{2N}.$$



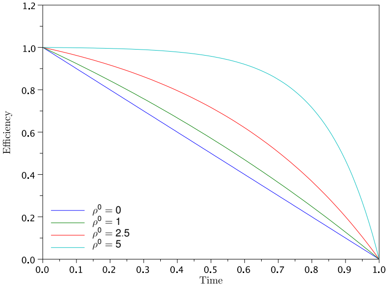
For a better comparison the resilience parameter is normalised by its counterpart when the market makers are risk neutral; $\sigma = 0.5$.

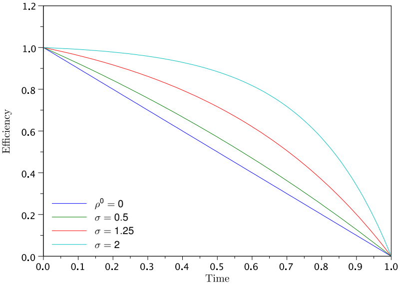
Efficiency

- Efficiency is a measurement of how informative the market prices are. Using the explicit form of the Ornstein-Uhlenbeck processes, we get

$$\Sigma(t) = \mathrm{Var}(\eta|\mathcal{F}_t^M) = \frac{1 - \exp\left(-\frac{\lambda\sigma^2\rho}{N}(1-t)\right)}{1 - \exp\left(-\frac{\lambda\sigma^2\rho}{N}\right)} \geq 1 - t.$$

- The loss of efficiency is monotone in ρ and σ. This is in contrast with the risk neutral case where the efficiency is independent of the volatility of the noise trading.





The straight line below the other curves is the efficiency when the market makers are risk neutral. In the left pane σ is taken to be 0.5 while $\rho^0 = 1$ on the right.

Price reversal

- When the market makers become risk averse, the prices exhibit a reversal. In particular,

$$M(s) := \lim_{\substack{t-s=\varepsilon \\ u-s=\varepsilon \\ \varepsilon\to 0}} \frac{\mathrm{Cov}\left(H(t,Y_t)-H(s,Y_s), H(u,Y_u)-H(t,Y_t)\right)}{\sqrt{\mathrm{Var}(H(t,Y_t)-H(s,Y_s))\mathrm{Var}(H(u,Y_u)-H(t,Y_t))}\varepsilon}$$

$$= -\frac{\rho\sigma^2\lambda}{4N}\left(1+\exp\left(-\frac{\lambda\sigma^2\rho}{N}s\right)\right).$$

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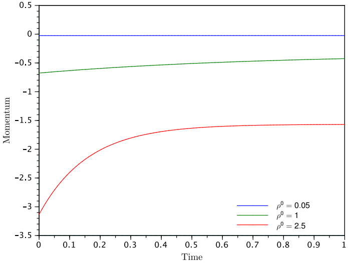
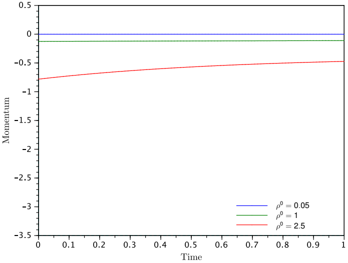
$$= -\frac{\rho\sigma^2\lambda}{4N}\left(1+\exp\left(-\frac{\lambda\sigma^2\rho}{N}s\right)\right).$$

- Similar calculations yield that when a strategic trader has no private information

$$M(s) = -\frac{\lambda^0\mu}{2}\left(1+\exp\left(-\lambda^0\sigma^2\rho^0 s\right)\right),$$

where $\rho^0 := \frac{\rho}{N}$, $\lambda^0 = \frac{\rho}{4N}$, and $\mu = \frac{\sigma^2\rho}{2N}$.





Both plots above assume $\sigma = 1$, and show monotonic behaviour of price reversal as a function of time and risk aversion. The right pane plots price reversal as a function of time and risk aversion for the case of strategic trader equilibrium, whereas the left one plots it for the case of the insider equilibrium

Insider's profits

- The ex-ante profit of the insider is found to be $\frac{1+\sigma^2\lambda^2}{2\lambda}$. Thus, the change in the ex-ante profits due to the risk aversion is equal to $\frac{(1-\lambda\sigma)^2}{2\lambda} > 0$.

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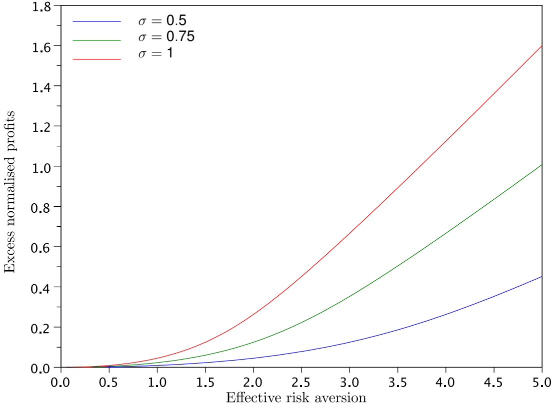
- The excess profits for the insider is monotonically increasing to infinity in both risk aversion and noise volatility. This implies that the noise traders lose more in the equilibrium with higher risk aversion of market makers.



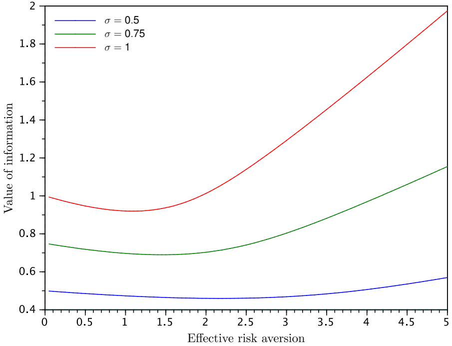
Insider's profits increase as the market makers get more risk averse.

Excess normalised profits, measured by $\frac{(1-\sigma\lambda)^2}{2\lambda\sigma}$, increase, too.

Value of information

The difference between the ex-ante profit of the insider and that of the strategic trader is the value of information and it equals to

$$\frac{1}{2\lambda} + \frac{\sigma^2}{2}(\lambda - \lambda^0).$$



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- non-systematic changes in price sensitivity to the total order.

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