Robust maximization problem with non-entropic penalty term

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Outline

Motivations

We study a Robust maximization problem with non entropic term in two cases :

1. **f-divergence penalty studied in the general framework of a continuous filtration.**
2. **consistent time penalty studied in the context of a Brownian filtration.**

- F. Wahid, **A.M**, M., Mnif : Robust utility maximization with a general penalty term. arXiv :1302.0442 (2013).

Outline

We present a problem of utility maximization under model uncertainty :

$$\sup_{\pi} \inf_{Q} U(\pi; Q)$$

where

- π runs through a set of strategies (portfolios, investment, decisions,...)
- Q runs through a set of models $\mathcal{Q}$.

Various Approaches

- **HJB approach** : Anderson, Hansen and Sargent (2003)
- **Duality approach** : Schied and Wu (2005),H. Follmer and A. Gundel (2005), Schied(2007),
- **BSDE approach** :
  Bordigoni, M. and Schweizer (2007)
  Lazrak-Quenez (2003), Quenez (2004) Duffie and Epstein (1992), Duffie and Skiadas (1994), Skiadas (2003), Schroder and Skiadas (1999, 2003, 2005)
  Laeven and Stadje (2012)
- **Risk measure** : Jouini, Schachermayer and Touzi (2006), Barrieu and El Karoui (2005)

Example : Skiadas (1999), Skiadas and Schroder (2001)

- Let us consider an agent with time-additive expected utility over consumptions paths :

$$\mathbb{E}\left[\int_0^T e^{-\delta t} u(c_t) dt\right].$$

with respect to some model $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P}, (B_t)_{t \geq 0})$ where $(B_t)_{t \geq 0}$ is Brownian motion under $\mathbb{P}$.

- Suppose that the agent has some preference to use another model $\mathbb{P}^\theta$ under which :

$$B_t^\theta = B_t - \int_0^t \theta_s ds$$

is a Brownian motion.

Example

- The agent evaluate the distance between the two models in term of the relative entropy of $\mathbb{P}^\theta$ with respect to the reference measure $\mathbb{P}$:

$$\mathcal{R}^\theta = \mathbb{E}^\theta \Big[\int_0^T e^{-\delta t} |\theta_t|^2 dt \Big]$$

- In this example, our robust control problem will take the form :

$$V_0 := \inf_\theta \Big[\mathbb{E}^\theta \Big[\int_0^T e^{-\delta t} u(c_t) dt \Big] + \beta \mathcal{R}^\theta \Big].$$

- The answer of this problem will be that : $V_0 = Y_0$ where Y is solution of BSDE or recursion equation :

$$Y_t = \mathbb{E}\Big[\int_t^T e^{-\delta(s-t)} \big(u(c_s) ds - \frac{1}{2\beta} d\langle Y \rangle_s \big) \Big| \mathcal{F}_t \Big],$$

Outline

Entropic case : semimartingale setting

- Bordigoni, G. , M.A. and Schweizer, M. (2007) have studied the following problem :

$$\inf_{Q \in \mathcal{Q}_f} E_Q[\mathcal{U}_{0,T} + \beta \mathcal{R}_{0,T} := \inf_{Q \in \mathcal{Q}_f} \Gamma(Q)$$

where

- T a finite time horizon.
- $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ filtered space under usual conditions.
- Possible scenarios given by

$$\mathcal{Q}_f := \{Q \ll P \; Q = P \;\; on \;\; \mathcal{F}_0, \quad H(Q|P) = E_Q[\ln(Z_T^Q)] < \infty\}$$

- The density process of Q with respect to P is the RCLL $P-$martingale

$$Z_t^Q = \frac{dQ}{dP}|_{F_t} = E_P[\frac{dQ}{dP}|F_t]$$

- β a non negative constant : the strength of this penalty term.

- The utility term :

$$\mathcal{U}_{t,T} = \alpha \int_t^T \frac{S_s^\delta}{S_t^\delta} U_s ds + \bar{\alpha} \frac{S_T^\delta}{S_t^\delta} \bar{U}_T$$

where :

  - α and $\bar{\alpha}$ are non negative constants
  - $S^\delta = (S_t^\delta := e^{-\int_0^t \delta_u du})_{0 \leq t \leq T}$ discount factor
  - $U = (U_t)_{0 \leq t \leq T}$ unbounded progressively measurable process corresponding : the utility rate process which comes from consumptions.
  - $\bar{U}_T$ is a unbounded $\mathcal{F}_T$-measurable random variable : the terminal utility at time T which corresponds to final wealth.

- The penalty term :

$$\mathcal{R}_{t,T}(Q) = \frac{1}{S_t^\delta} \int_t^T \delta_s S_s^\delta \ln(\frac{Z_s^Q}{Z_t^Q}) ds + \frac{S_T^\delta}{S_t^\delta} \ln(\frac{Z_T^Q}{Z_t^Q}).$$

The case $\delta = 0$

The spacial case $\delta = 0$ corresponds to the cost functional
$\Gamma(Q) = E_Q[\mathcal{U}_{0,T}] + \beta H(Q|P) = \beta H(Q|P_{\mathcal{U}}) - \beta \ln E_P[\exp(-\frac{1}{\beta}\mathcal{U}_{0,T})]$
Where $P_{\mathcal{U}} \approx P$ and $\frac{dP_{\mathcal{U}}}{dP} = c \exp(-\frac{1}{\beta}\mathcal{U}_{0,T})$

- Csiszar (1975) have proved the existence and uniqueness of the optimal measure $Q^* \approx P_{\mathcal{U}}$ which minimize the relative entropy $H(Q|P_{\mathcal{U}})$.
- I. Csiszar : I-divergence geometry of probability distributions and minimization problems. Annals of Probability 3, p. 146-158 (1975).

Functional spaces

- $L^{\exp}$ is the space of all $\mathcal{F}_T-$ measurable random variables X with $E_P[\exp(\gamma X)] < +\infty \;\; \forall \;\; \gamma > 0$.
- $D_0^{\exp}$ is the space of all progressively measurable processes $X = (X_t)_{0\leq t\leq T}$ with : $E_P\Big[\exp(\gamma \text{ess} \sup_{0\leq t\leq T} |X_t|)\Big] < \infty \;\; \forall \;\; \gamma > 0$.
- $D_1^{\exp}$ is the space of all progressively measurable processes $X = (X_t)_{0\leq t\leq T}$ such that : $E_P\Big[\exp(\gamma \int_0^T |X_s|ds)\Big] < \infty \; \forall \; \gamma > 0$.
- $\mathcal{M}_0^p(P)$ is the space of all $\mathbb{P}-$ martingales $M = (M_t)_{0\leq t\leq T}$ such that $M_0 = 0$ and

$$E_P[\sup_{0\leq t\leq T} |M_t|^p] < +\infty.$$

Existence result for the entropic case

We define

$$V_t = ess \inf_{Q \in \mathcal{Q}_f} \left(E_Q[\mathcal{U}_{t,T} + \beta \mathcal{R}_{t,T}(Q)] \right).$$

Assumptions

(H1) $0 \leq \delta_t \leq \|\delta\|_\infty$ for some constant $\|\delta\|_\infty$
(H2) $U \in D_1^{\exp}$ and $\bar{U} \in L^{\exp}$
(H3) The filtration $\mathbb{F}$ is continuous

Existence result for the entropic case

Theorem (Bordigoni, A. M. and Schweizer)

1. There exist a unique Q^* which minimizes $Q \mapsto \Gamma(Q)$ aver all $Q \in \mathcal{Q}_f$.
2. The optimal measure Q^* is equivalent to P.
3. The couple (V, M^V) is the unique solution in $D_0^{\exp} \times \mathcal{M}_0^p(P)$ of the BSDE :

$$\begin{cases} dV_t = (\delta_t V_t - \alpha U_t)dt + \frac{1}{2\beta} d\langle M^V \rangle_t + dM_t^V \\ V_T = \bar{\alpha}\bar{U} \end{cases}$$

and the density of the probability measure Q^* is given by $Z_t^{Q^*} = \mathcal{E}(-\frac{1}{\beta} M_t^V)$

Existence and uniqueness of solution for BSDE

- Existence : based on the Martingale optimality principle
- Uniqueness : based on the recursive relation

$$V_t = -\beta \ln \mathbb{E}_Q[\exp(-\frac{1}{\beta}\int_t^T (\alpha U_s - \delta_s V_s) ds) - \frac{1}{\beta}\bar{\alpha}\bar{U}_T | \mathcal{F}_t]$$

Quadratic BSDE with unbounded terminal condition

- Skiadas, Schröder (2001)
- Briand and Hu (2007, 2009)
- Barrieu and El Karoui N (2010) : Forward approach based on quadratic semimartingale.
- El Karoui, M., Ngoupeyou. Quadratic BSDE with jumps and unbounded terminal condition. Preprint 2012.

Outline

The case of f-divergence penalty

- The cost functional :

$$c(\omega, Q) := \mathcal{U}^{\delta}_{0,T} + \beta \mathcal{R}^{\delta}_{0,T}(Q)$$

$$\inf_{Q \in \mathcal{Q}} E_Q[\mathcal{U}_{0,T} + \beta \mathcal{R}_{0,T}(Q)]$$

where

- $$\mathcal{U}^{\delta}_{t,T} := \alpha \int_t^T S^{\delta}_s U_s ds + \bar{\alpha} S^{\delta}_T \bar{U}_T$$

- $\mathcal{R}_{t,T}(Q)$ is a penalty term which is written as a sum of a penalty rate and a final penalty given by :

$$\mathcal{R}^{\delta}_{0,T} := \int_0^T \delta_s S^{\delta}_s \frac{f(Z^Q_s)}{Z^Q_s} ds + S^{\delta}_T \frac{f(Z^Q_T)}{Z^Q_T}, \quad \text{for all } 0 \le t \le T$$

Class of f-divergence penalty

where $f : [0, +\infty) \mapsto \mathbb{R}$ is continuous, strictly convex and satisfies the following assumptions :

assumption

(H.1) $f(1) = 0$.

(H.2) There is a constant $\kappa \in \mathbb{R}_+$ such that $f(x) \geq -\kappa, \quad \text{for all} \quad x \in (0, +\infty)$.

(H.3) $\lim\limits_{x \mapsto +\infty} \dfrac{f(x)}{x} = +\infty$.

Class of f-divergence penalty

- Our basic goal is to

$$\text{minimize the functional } Q \mapsto \Gamma(Q) := E_Q[c(., Q)]$$

over a suitable class of probability measures $Q \ll P$ on $\mathcal{F}_T$.

- We define the conjugate function of f on $\mathbb{R}_+$ by :

$$f^*(x) := \sup_{y>0} (xy - f(y)).$$

Functional spaces

- L^{f^*} is the space of all $\mathcal{F}_T$ measurable random variables X with

$$E_P\left[f^*\left(\gamma|X|\right)\right] < \infty \qquad \text{for all } \gamma > 0,$$

- $D_0^{f^*}$ is the space of all progressively measurable processes $X = (X_t)_{0\le t\le T}$ with

$$E_P\left[f^*\left(\gamma \operatorname{ess\,sup}_{0\le t\le T}|X_t|\right)\right] < \infty \qquad \text{for all } \gamma > 0,$$

- $D_1^{f^*}$ is the space of all progressively measurable processes $X = (X_t)_{0\le t\le T}$ such that

$$E_P\left[f^*\left(\gamma\int_0^T|X_s|ds\right)\right] < \infty \qquad \text{for all } \gamma > 0.$$

f-divergence

Definition

For any probability measures Q on $(\Omega, \mathcal{F})$, we define the f-divergence of Q with respect to P by :

$$d(Q|P) := \begin{cases} E_P[f(\frac{dQ}{dP}|_{\mathcal{F}_T})] & \text{if } Q \ll P \text{ on } \mathcal{F}_T \\ +\infty & \text{otherwise} \end{cases} .$$

- We denote by $\mathcal{Q}_f$ the space of all probability measures Q on $(\Omega, \mathcal{F})$ with $Q \ll P$ on $\mathcal{F}_T$, $Q = P$ on $\mathcal{F}_0$ and $d(Q|P) < +\infty$.
- The set $\mathcal{Q}_f^e$ is defined as follows

$$\mathcal{Q}_f^e := \{Q \in \mathcal{Q}_f | Q \approx P \text{ on } \mathcal{F}_T\}.$$

Assumption

(A1) The process δ is positive and bounded by $\|\delta\|_\infty$.
(A2) The process U belongs to $D_1^{f^*}$ and the random variable $\bar{U}_T$ is in L^{f^*}

Remark

In the case of entropic penalty, we have $f(x) = x\ln(x)$ and then $f^*(x) = \exp(x-1)$. As in Bordigoni, M. and Schweizer (2007), the integrability conditions are formulated as

$$\mathbb{E}_P\left[\exp(\gamma\int_0^T |U(s)|ds)\right] < +\infty \text{ and } \mathbb{E}_P\left[\exp(\gamma|\bar{U}_T|)\right] < +\infty\ \forall\gamma > 0$$

Existence of optimal probability measure

proposition

Under **(A1)-(A2)**,for all $Q \in \mathcal{Q}$; we have

1. $c(.,Q) \in L^1(Q)$
2. $\Gamma(Q) \leq C(1 + d(Q|P))$ for some a constant $C \in (0, +\infty)$ which depends only on $\alpha, \bar{\alpha}, \beta, \delta, T, U, \bar{U}$.
3. There exists a positive constant K which depends only on $\alpha, \bar{\alpha}, \beta, \delta, T, U, \bar{U}$ such that

$$d(Q|P) \leq K(1 + \Gamma(Q)).$$

   In particular $\inf_{Q \in Q_f} \Gamma(Q) > -\infty.$

In particular $\Gamma(Q)$ is well-defined and finite for every $Q \in \mathcal{Q}_f$

Existence of optimal probability measure

Theorem

*Under **(A1)-(A2)**, there exists a unique* $Q^* \in \mathcal{Q}_f$ *which minimizes* $Q \mapsto \Gamma(Q)$ *over all* $Q \in \mathcal{Q}_f$.

(A3) f is differentiable on $(0, +\infty)$ and $f'(0) = \lim_{x \to 0^+} f'(x) = -\infty$.

Theorem

Under the Assumptions $(A1)$*-*$(A3)$*, the optimal probability measure* Q^* *is equivalent to* P.

Bellman optimality principle

Let $\mathcal{S}$ denote the set of all $\mathcal{F}$-stopping times τ with values in $[0, T]$ and $\mathcal{D}$ the space of all density processes Z^Q with $Q \in \mathcal{Q}_f$. We define

$$\mathcal{D}(Q, \tau) := \{Z^{Q'} \in \mathcal{D}; Q = Q' \text{ on } \mathcal{F}_\tau\}$$

$$\Gamma(\tau, Q) := E_Q[c(., Q)|\mathcal{F}_\tau]$$

and the minimal conditional cost at time τ ,

$$J(\tau, Q) := Q \text{ - } \operatorname*{essinf}_{Q' \in \mathcal{D}(Q,\tau)} \Gamma(\tau, Q').$$

Then

$$\inf_{Q \in \mathcal{Q}_f} \Gamma(Q) = \inf_{Q \in \mathcal{Q}_f} E_Q[c(., Q)] = E_P[J(0; Q)]$$

Proposition (Bellman optimality principle)

1. The family $\{J(\tau, Q) | \tau \in \mathcal{S}, Q \in \mathcal{Q}_f\}$ is a submartingale system.
2. $Q^* \in \mathcal{Q}_f$ is optimal $\Leftrightarrow$ $\{J(\tau, Q^*) | \tau \in \mathcal{S}\}$ is a martingale system.
3. For all $Q \in \mathcal{Q}_f$ there is an adapted RCLL process $J^Q = (J_t^Q)_{0 \leq t \leq T}$ which is a right closed Q-submartingale such that : $J_\tau^Q = J(\tau, Q)$ Q-a.s for each stopping time τ.

Recursive representation and BSDE ? : still open questions ?

Outline

Setting

- $(\mathcal{F}_t)_{0\leq t\leq T}$ is generated by a d-dimensional Brownian motion W. $Q \ll P$ on $\mathcal{F}_T$.
- The density process of Q with respect to P is a RCLL martingale $Z^Q = (Z_t^Q)_{0\leq t\leq T}$ given by :

$$Z_t^Q = \mathcal{E}\left(\int_0^t \eta_u dW_u\right) \;\; Q.p.s, \forall t \in [0, T].$$

- We consider a deterministic function h defined on $\mathbb{R}^d$ such that there are two positive constants κ_1 and κ_2 satisfying :

$$h(x) \geq \kappa_1 \|x\|^2 - \kappa_2.$$

- The penalty term is defined by

$$\mathcal{R}^{\delta}_{t,T}(Q^{\eta}) = \int_t^T \delta_s \frac{S^{\delta}_s}{S^{\delta}_t}\left(\int_t^s h(\eta_u)du\right)ds + \frac{S^{\delta}_T}{S^{\delta}_t}\int_t^T h(\eta_u)du.$$

for $Q \ll P$ on $\mathcal{F}^W_T$.

- As in the case of f-divergence penalty, we have to solve the following optimization problem :

$$\text{minimize the functional } Q^{\eta} \mapsto \Gamma(Q^{\eta}) := E_{Q^{\eta}}[c(., Q^{\eta})]$$

Definition

For all probability measure Q^η on $(\Omega, \mathcal{F})$, we define the penalty function :

$$\gamma_t(Q^\eta) := \begin{cases} E_{Q^\eta}[\int_t^T h(\eta_s)ds|\mathcal{F}_t] & \text{if } Q^\eta \ll P \text{ on } \mathcal{F}_T \\ +\infty & \text{otherwise} \end{cases} .$$

We note $\mathcal{Q}_f^c$ the space of all probability measures Q^η on $(\Omega, \mathcal{F})$ such that $Q^\eta \ll P$ on $\mathcal{F}_T$ and $\gamma_0(Q^\eta) < +\infty$ and $\mathcal{Q}_f^{c,e} := \{Q^\eta \in \mathcal{Q}_f | Q \approx P \,\text{on}\, \mathcal{F}_T\}$.

Remark

1. We note that $\mathcal{Q}_f^{c,e}$ is non empty set because $P \in \mathcal{Q}_f^{c,e}$.
2. The particular case of $h(x) = \frac{1}{2}|x|^2$ corresponds to the entropic penalty.
3. For a general function h we have for all $Q^\eta \in \mathcal{Q}_f^c$,

$$H(Q^\eta|P) \leq \frac{1}{2\kappa_1}\gamma_0(Q^\eta) + \frac{T\kappa_2}{2\kappa_1}.$$

Assumption

(A'2) : the cost process U belongs to $D_1^{\exp}$ and the terminal target $\bar{U}$ is in $L^{\exp}$

Remark

Under Assumption **(A'2)**, we have

$$\lambda \int_0^T |U_s| ds + \mu |\bar{U}_T| \in L^{\exp}, \quad \text{for all} \quad (\lambda, \mu) \in \mathbb{R}_+^2.$$

Theorem

Assume that ***(A1)-(A'2)*** *are satisfied. Then there exists a probability measure* $Q^{\eta^*} \in \mathcal{Q}_f^c$ *minimizing* $Q^\eta \mapsto \Gamma(Q^\eta)$ *over all* $Q^\eta \in \mathcal{Q}_f^c$.

Theorem

Under the assumptions ***(A1)-(A'2)****, the pair* (V, Z) *is the unique solution in* $D_0^{\exp} \times \mathcal{H}_d^p, p \geq 1$*, of the following BSDE :*

$$\begin{cases} dY_t = (\delta_t Y_t - \alpha U_t + h^*(\frac{1}{\beta} Z_t))dt - Z_t dW_t, \\ Y_T = \alpha' U'_T. \end{cases}$$

and Q^* *is equivalent to* P.

Comparison with related result

- In the case of the entropic penalty, which corresponds to $h(x) = \frac{1}{2}|x|^2$, the value process is described through the backward stochastic differential equation :

$$\begin{cases} dY_t = (\delta_t Y_t - \alpha U_t + \frac{1}{2\beta}|Z|_t^2)dt - Z_t dW_t \\ \quad Y_T = \alpha' U'_T \end{cases} .$$

  These results are obtained by Schroder and Skiadas (2003) where $\alpha' = 0$.

- Dynamic concave utility : Delbaen, Hu and Bao (2009) treated the case $\delta = 0$ and $\xi = \alpha' U'$ is bounded and $\beta = 1$.

Comparison with related result

- In this special case the existence of an optimal probability was shown by Jouini, Schachermayer and Touzi's work (2005).
- Delbaen et al. showed (using a different method) that the dynamic concave utility

$$Y_t = ess \inf_{Q \in \mathcal{Q}_f} E[\xi + \int_t^T h(\eta_u) du | \mathcal{F}_t]$$

satisfies the following BSDE :

$$\begin{cases} dY_t = h^*(Z_t)dt - Z_t dW_t \\ Y_T = \xi \end{cases}.$$

Outline

The model

- We consider a filtered probability space $(\Omega, \mathcal{G}, \mathbb{G}, \mathbb{P})$. All the processes are taken $\mathbb{G}$-adapted, and are defined on the time interval $[0, T]$.
- Any special $\mathbb{G}$-semimartingale Y admits a canonical decomposition $Y = Y_0 + A + M^{Y,c} + Y^{Y,d}$ where A is a predictable finite variation process, Y^c is a continuous martingale and $M^{Y,d}$ is a pure discontinuous martingale.
- For each $i = 1, \ldots, n$, H^i is a counting process and there exist a positive adapted process λ^i, called the $\mathbb{P}$ intensity of H^i, such that the process N^i with $N^i_t := H^i_t - \int_0^t \lambda^i_s ds$ is a martingale.
- We assume that the processes $H^i, i = 1, \ldots, d$ have no common jumps.

The model

- Any discontinuous martingale admits a representation of the

$$dM_t^{Y,d} = \sum_{i=1}^{d} \hat{Y}_t^i dN_t^i$$

where $\hat{Y}^i, i = 1, \ldots, d$ are predictable processes.

Semimartingale BSDE with jumps

Definition

A solution of the BSDE is a triple of processes $(Y, M^{Y,c}, \widehat{Y})$ such that Y is a P-semimartingale, M is a locally square-integrable locally martingale with $M_0 = 0$ and $\widehat{Y} = (\widehat{Y}^1, \cdots, \widehat{Y}^d)$ a $\mathbb{R}^d$-valued predictable locally bounded process such that :

$$\begin{cases} dY_t = [\sum_{i=1}^{d} g(\widehat{Y}_t^i)\lambda_t^i - U_t + \delta_t Y_t]dt + \frac{1}{2}d\langle M^{Y,c}\rangle_t + dM_t^{Y,c} \\ + \sum_{i=1}^{d} \widehat{Y}_t^i dN_t^i \\ Y_T = \bar{U}_T \end{cases} \quad (1)$$

where $g(x) = e^{-x} + x - 1$.

Existence result

Theorem (Jeanblanc, M., M. A., Ngoupeyou A.)

- *There exists a unique triple of process* $(Y, M^{Y,c}, \widehat{Y}) \in D_0^{\exp} \times \mathcal{M}_{0,loc}(P) \times \mathcal{L}^2(\lambda)$ *solution of the semartingale BSDE with jumps.*
- *Furthermore, the optimal measure* $\mathbb{Q}^*$ *solution of our minimization problem is given :*

$$dZ_t^{\mathbb{Q}^*} = Z_{t^-}^{\mathbb{Q}^*} dL_t^{\mathbb{Q}^*}, \quad Z_0^{\mathbb{Q}^*} = 1$$

where

$$dL_t^{\mathbb{Q}^*} = -dM_t^{Y,c} + \sum_{i=1}^{d} \left(e^{-\widehat{Y}_t^i} - 1 \right) dN_t^i.$$

The model :example from Credit Risk

Example

- *We assume that* $\mathbb{G}$ *is the filtration generated by a continuous reference filtration* $\mathbb{F}$ *and* d *positive random times* $\tau_1, \cdots, \tau_d$ *which are the default times of* d *firms :* $\mathbb{G} = (\mathcal{G}_t)_{t \geq 0}$ *where*

$$\mathcal{G}_t = \bigcap_{\epsilon > 0} \mathcal{F}_{t+\epsilon} \vee \sigma(\tau_1 \wedge t + \epsilon) \vee \sigma(\tau_2 \wedge t + \epsilon) \cdots \vee \sigma(\tau_d \wedge t + \epsilon)$$

  where $\sigma(\tau_i \wedge t + \epsilon)$ *is the generated* σ*-fields which is non random before the default times* τ_i *for each* $i = 1, \cdots, d$.
- *we note* $H_t^i = \mathbf{1}_{\{\tau_i \leq t\}}$.

The model :example from Credit Risk

Example

- *We assume that each* τ_i *is* $\mathbb{G}$*-totaly inaccessible and there exists a positive* $\mathbb{G}$*-adapted process* λ^i *such that, the process* N^i *with* $N^i_t := H^i_t - \int_0^t \lambda^i_s ds$ *is a* $\mathbb{G}$*-martingale.*
- *Obviously, the process* λ^i *is null after the default time* τ_i*.*

The model :example from Credit Risk

Example

- *From Kusuoka, the representation of the discontinuous martingale $M^{Y,d}$ with respect to N^i holds true when the filtration $\mathbb{G}$ is generated by a Brownian motion and the default processes under (H) hypothesis.*

Questions ?

Thank you for your attention !