The Pricing Model of Corporate Securities under Cross-Holdings of Equities and Debts

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Introduction

Focus

Systemic risk in financial markets and pricing corporate securities

1. Payoff values under cross-holdings (-ownerships) of corporate securities.
   - Determination of payoff values is not clear
   - Why?
2. No-arbitrage prices of corporate securities under cross-holdings

Motivation: Why cross-holdings of securities is important?

- Because the situation of EU countries can be seen as a kind of cross-holdings of debts after the integration of EU and the currency unification.
- Suppose that outside EU investors hold Greek government bonds and German government bonds.
- Before currency unification, investing in Greek and German bonds could be a kind of diversified investment.
- But now, German banks hold Greek government bonds and Greek banks also hold German government bonds.
- Beside this, both bonds are issued by the same currency.
- *Outside EU investors need to develop the new theory for investing in EU countries.*

Literature review (1/4)

Stability of the financial system (cross-holdings of debts):

- Eisenberg and Noe (2001):
  - They call a set of payoffs clearing (payment) vector.
  - They call the determination of (an algorithm to derive) the payoffs clearing mechanism.
  - greatest clearing vector $\overline{\mathbf{p}}$, least clearing vector $\underline{\mathbf{p}}$
  - Under regular condition without default costs, $\overline{\mathbf{p}} = \underline{\mathbf{p}}$.
- Rogers and Veraart (2012) consider default costs. $\overline{\mathbf{p}} \neq \underline{\mathbf{p}}$.

Literature review (2/4)

Pricing of securities (cross-holdings of stock and debts):

- Suzuki (2002): cross-holdings of debts and equities.
  - Contraction mapping algorithm
- Fischer (2012): extends with bond seniority structure and derivatives.

We extend Fischer (2012) considering default costs and introducing early clearing vector $\overline{\mathbf{q}}, \underline{\mathbf{q}}$

Literature review (3/4)

Structural Models

- Merton (1974):

Firm i

$\tilde{e}_i$ business asset	$p_i^1 = \tilde{e}_i \wedge b_i$ debt
	$p_i^0 = (\tilde{e}_i - b_i)_+$ equity

- Pricing formula:

$$\begin{aligned} &\text{equity} \quad v_i^0 = \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} \max\{\tilde{e}_i - b_i, 0\}], \\ &\text{debt} \quad v_i^1 = \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} \min\{b_i, \tilde{e}_i\}]. \end{aligned}$$

Literature review (4/4)

Structural Models (cont.)

- Suzuki (2002): Balance Sheets at Maturity

Firm i

	$\tilde{e}_i$	p_i^1
financial assets	$\pi_{ij}^1 p_j^1$	p_i^0
	$\pi_{ij}^0 p_j^0$	

Firm j

$\tilde{e}_j$	p_j^1
$\pi_{ji}^1 p_i^1$	p_j^0
$\pi_{ji}^0 p_i^0$	

- There is a financial feedback loop among firms.
- Note:

$$p_i^1 = (\tilde{e}_i + \pi_{ij}^1 p_j^1 + \pi_{ij}^0 p_j^0) \wedge b_i, \qquad p_j^1 = (\tilde{e}_j + \pi_{ji}^1 p_i^1 + \pi_{ji}^0 p_i^0) \wedge b_j,$$

$$p_i^0 = (\tilde{e}_i + \pi_{ij}^1 p_j^1 + \pi_{ij}^0 p_j^0 - b_i)^+, \qquad p_j^0 = (\tilde{e}_j + \pi_{ji}^1 p_i^1 + \pi_{ji}^0 p_i^0 - b_j)^+.$$

Our study

What we do in this study:

- Extend Fischer (2012) by considering default costs.
- Introduce early clearing vector $\underline{\mathbf{q}}, \overline{\mathbf{q}}$ to capture the financial crisis.

Main results:

- Prove the existence of $\overline{\mathbf{p}}, \underline{\mathbf{p}}$ and $\underline{\mathbf{q}}$ with default costs and with bonds' seniority structure.
- Using numerical examples
    - $\underline{\mathbf{p}}$ is not so serious for the economy.
    - $\underline{\mathbf{q}}$ can capture the credit spreads under financial crisis.
    - Traditional Merton model underestimates the credit risk of debts when we consider cross holding of debts with default costs.

Model Setup

Notations

The following notations are used in this talk:

lower	x, y etc.	scalars
lower and bold	$\mathbf{x}$, $\mathbf{y}$, etc.	vectors
upper and bold	$\mathbf{X}$, $\mathbf{Y}$, etc.	matrices

$$\mathbf{0} = (0, \dots, 0)^\top, \quad \mathbf{1} = (1, \dots, 1)^\top, \quad \mathbf{O} = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1 \end{pmatrix},$$

$$\mathbf{x} \wedge \mathbf{y} = \begin{pmatrix} \min\{x_1, y_1\} \\ \vdots \\ \min\{x_n, y_n\} \end{pmatrix}, \quad \mathbf{x} \vee \mathbf{y} = \begin{pmatrix} \max\{x_1, y_1\} \\ \vdots \\ \max\{x_n, y_n\} \end{pmatrix}, \quad (\mathbf{x})_+ = \mathbf{x} \vee \mathbf{0},$$

Financial markets

- There are totally n firms in the financial market.
- They cross-hold their equities and debts.
- e_i: the business (external) asset of firm i at maturity.

$$\mathbf{e} = (e_1, \ldots, e_n)^\top \in \boldsymbol{R}_+^n.$$

- The realization of $\mathbf{e}$ is independent of capital structures of the firms and the structures of cross-holdings.

Debts

- The liability of firm i has a seniority structure with at most m priorities.
- b_i^k: the face value of k-th subordinated debt issued by firm i, $i = 1, \ldots, n$ and $k = 1, \ldots, m$.

$$\mathbf{b}^k = (b_1^k, \ldots, b_n^k)^\top \in \boldsymbol{R}_+^n.$$

- Define d_i^k by

$$d_i^k = \sum_{\ell=k+1}^{m} b_i^\ell$$

and

$$\mathbf{d}^k = (d_1^k, \ldots, d_n^k)^\top \in \boldsymbol{R}_+^n.$$

Cross-holding structure and payoffs

- $\pi_{ij}^k \in [0,1]$: proportion of firm i's ownership of b_j^k.
- $\mathbf{\Pi}^k = (\pi_{ij}^k)_{i,j=1}^n$: cross-holding structure of k-th subordinated debt ($k = 1, \ldots, m$).
- $\mathbf{\Pi}^0 = (\pi_{ij}^0)_{i,j=1}^n$: cross-holding structure of the equities.
- p_i^k: the payoff of firm i's k-th debt ($k = 1, \ldots, m$)
- p_i^0: the payoff of firm i's equity.

$$
\begin{aligned}
\mathbf{p} &= (p_1^0, \ldots, p_n^0, p_1^1, \ldots, p_n^1, \ldots, p_1^m, \ldots, p_n^m)^\top \in \boldsymbol{R}^{(m+1)n} \\
&= ((\mathbf{p}^0)^\top, (\mathbf{p}^1)^\top, \ldots, (\mathbf{p}^m)^\top)^\top
\end{aligned}
$$

- We call $\mathbf{p}$ payment vector.

Default costs

- a_i: The total asset of firm i at maturity before liquidation.

$$a_i(\mathbf{p}) = e_i + \sum_{\ell=0}^{m} \sum_{j=1}^{n} \pi_{ij}^{\ell} p_j^{\ell}.$$

- Default of firm i is determined by the following inequality:

$$a_i(\mathbf{p}) < d_i^0.$$

- If firm i defaults, the payment resource for the debts is reduced to $(1 - \delta_i)a_i$.
- $\delta_i \in [0, 1]$ describes the proportional cost associated with the default.
- $\boldsymbol{\Delta} = \mathrm{diag}[\delta_i]$.

Definition of clearing payment vector (1)

Definition

We say that $\mathbf{p} \in \boldsymbol{R}_+^{(m+1)n}$ is a clearing payment vector if

$$p_i^0 = \left(a_i(\mathbf{p}) - d_i^0\right)_+$$

and

$$p_i^k = 1_{\{a_i(\mathbf{p}) \geq d_i^0\}} b_i^k + 1_{\{a_i(\mathbf{p}) < d_i^0\}} \left(b_i^k \wedge \left((1-\delta_i)a_i(\mathbf{p}) - d_i^k\right)_+\right)$$

for $i = 1, \ldots, n$ and $k = 1, \ldots, m$.

Definition of clearing payment vector (2)

In matrix form,

$$\mathbf{p}^0 = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+$$

and

$$\mathbf{p}^k = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p})\left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k\right)_+\right),$$

for $k = 1, \dots, m$ where

$$\mathbf{a}(\mathbf{p}) = \mathbf{e} + \sum_{\ell=0}^{m} \boldsymbol{\Pi}^\ell \mathbf{p}^\ell,$$

$$\mathbf{D}(\mathbf{p}) = \mathrm{diag}\left[1_{\{a_i(\mathbf{p}) < d_i^0\}}\right].$$

We call **D** default matrix. Recall that

$$\mathbf{p} = \begin{pmatrix} \mathbf{p}^0 \\ \vdots \\ \mathbf{p}^m \end{pmatrix}, \mathbf{p}^\ell = \begin{pmatrix} p_1^\ell \\ \vdots \\ p_n^\ell \end{pmatrix}, \ell = 0, \dots, m$$

Existence of Clearing Vector

Fixed point problem

- Define the function $\mathbf{f}: \boldsymbol{R}_+^{(m+1)n} \to \boldsymbol{R}_+^{(m+1)n}$ by

$$\mathbf{f}^0(\mathbf{p}) = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+,$$
$$\mathbf{f}^k(\mathbf{p}) = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p})\left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k\right)_+\right)$$

  for $k = 1, \ldots, m$.
- The clearing payment vector is expressed as a fixed point $\mathbf{f}(\mathbf{p}) = \mathbf{p}$.
- If $\boldsymbol{\Delta} = \mathbf{O}$, $\mathbf{f}$ is a contraction mapping with respect to l^1-norm (Suzuki, 2002; Fischer, 2012). However, it is not in the case $\boldsymbol{\Delta} \neq \mathbf{O}$.

Existence of clearing payment vector and its Proof (1)

Proposition (Rogers and Veraart; 2012, Nishide and Suzuki; 2013)

Assume that $\mathbf{1}^\top \mathbf{\Pi}^0 < \mathbf{1}$. *Then, there exists a vector* $\mathbf{p} \in R_+^{(m+1)n}$ *such that* $\mathbf{f}(\mathbf{p}) = \mathbf{p}$.

Proof.

Let the vector $\mathbf{x} \in R_+^n$ *be given by the solution for*

$$\mathbf{x} = \left(\mathbf{e} + \mathbf{\Pi}^0 \mathbf{x} + \sum_{\ell=1}^{m} \mathbf{\Pi}^\ell \mathbf{b}^\ell - \mathbf{d}^0 \right)_+ .$$

We define the sequence of $(m+1)n$*-dimensional vectors* $\{\overline{\mathbf{p}}_h\}$ *by*

$$\overline{\mathbf{p}}_0 = (\mathbf{x}, \mathbf{b}^1, \ldots, \mathbf{b}^m)^\top \text{ and } \overline{\mathbf{p}}_h = \mathbf{f}(\overline{\mathbf{p}}_{h-1}) \text{ for } h \geq 1.$$

Proof of existence (2)

Proof (cont.)

(Sketch)

- *We can show that* $\{\overline{\mathbf{p}}_h\}_{h\geq 0}$ *is monotonically non-increasing.*
- *We can assure* $\{\overline{\mathbf{p}}_h\}$ *is bounded below* ($\overline{\mathbf{p}}_h \geq \mathbf{0}$).
- *Then,* $\{\overline{\mathbf{p}}_h\}$ *has a limit* $\overline{\mathbf{p}}$ *with* $\mathbf{f}(\overline{\mathbf{p}}) = \overline{\mathbf{p}}$.

Remarks of the proof

Remark

The sequence $\{\mathbf{p}_h\}$ is exactly the same as the algorithm proposed by Elsinger (2009). The proposition generalizes his result to the case where the default costs are present.

Remark

Alternatively, we can show the existence of a clearing vector with the sequence $\{\underline{\mathbf{p}}_h\}$ defined by

$$\underline{\mathbf{p}}_0 = \mathbf{0} \text{ and } \underline{\mathbf{p}}_h = \mathbf{f}(\underline{\mathbf{p}}_{h-1}).$$

The proof is completed by noticing that $\{\underline{\mathbf{p}}_h\}$ is a non-decreasing sequence and bounded above ($\underline{\mathbf{p}}_h \leq \overline{\mathbf{p}}_0$).

We denote the limit of $\{\underline{\mathbf{p}}_h\}$ by $\underline{\mathbf{p}}$.

Greatest and least clearing vectors

Proposition

The vector $\overline{\mathbf{p}}$ *is the greatest clearing vector in the sense that*

$$\mathbf{p} \leq \overline{\mathbf{p}}$$

for any $\mathbf{p} \in \boldsymbol{R}_+^{(m+1)n}$ *with* $\mathbf{f}(\mathbf{p}) = \mathbf{p}$. *Similarly the vector* $\underline{\mathbf{p}}$ *is the least clearing vector in the sense that*

$$\mathbf{p} \geq \underline{\mathbf{p}}$$

for any clearing vector $\mathbf{p} \in \boldsymbol{R}_+^{(m+1)n}$.

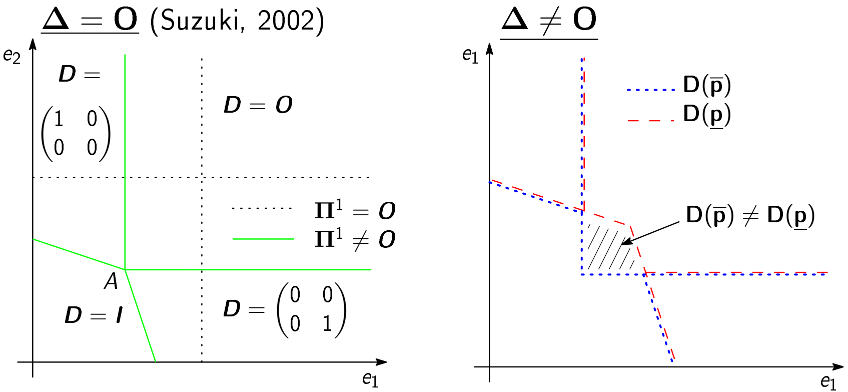
Notes:

- The proof can be given by contradiction.
- If $\boldsymbol{\Delta} = \mathbf{O}$, then $\underline{\mathbf{p}} = \overline{\mathbf{p}}$

Algorithm for $\overline{\mathbf{p}}$ and $\underline{\mathbf{p}}$

- Define the function $\mathbf{f}: \boldsymbol{R}_+^{(m+1)n} \to \boldsymbol{R}_+^{(m+1)n}$ by
  $$\mathbf{f}^0(\mathbf{p}) = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+,$$
  $$\mathbf{f}^k(\mathbf{p}) = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p})\left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k\right)_+\right)$$
  for $k = 1, \ldots, m$.
- If $\mathbf{p} = \mathbf{f}(\mathbf{p})$ then $\mathbf{p}$ is called a clearing payment vector.
- Greatest clearing payment vector is given by
  $$\overline{\mathbf{p}} = \lim_{h\to\infty} \overline{\mathbf{p}}_h \text{ where } \overline{\mathbf{p}}_0 = (\mathbf{x}, \mathbf{b}^1, \ldots, \mathbf{b}^m)^\top, \ \overline{\mathbf{p}}_h = \mathbf{f}(\overline{\mathbf{p}}_{h-1}).$$
- Least clearing payment vector is given by
  $$\underline{\mathbf{p}} = \lim_{h\to\infty} \underline{\mathbf{p}}_h \text{ where } \underline{\mathbf{p}}_0 = \mathbf{0}, \underline{\mathbf{p}}_h = \mathbf{f}(\underline{\mathbf{p}}_{h-1}).$$
- We can get $\{\overline{\mathbf{p}}, \mathbf{D}(\overline{\mathbf{p}})\}$ and $\{\underline{\mathbf{p}}, \mathbf{D}(\underline{\mathbf{p}})\}$ for $\exists\mathbf{e}$.

Illustration of D ($n = 2, m = 1, \Pi^0 = O$)



- When $\Delta \neq \mathbf{O}$, the region where $\mathbf{D} = \mathbf{I}$ increases, and the region $\overline{\mathbf{p}} \neq \underline{\mathbf{p}}$ comes out.

Motivation behind an early clearing

- In a normal situation, the price of even bad debts is given by considering they might be paid back at their face values.
- However, in a severe recession, people tend to sell their holding bad debts at a sacrifice.
- In this situation, the price of bad debts comes close to their recovery values.
- We will examine the situation where every firm values their holding debts at their recovery values.
- In Merton's setting:

$$q^1 = \min\{(1-\delta)a, b\}, \quad q^0 = 1_{\{(1-\delta)a>b\}} \max\{a-b, 0\}$$

Definition of early clearing payment vector (1)

Definition

We say that $\mathbf{q} \in \boldsymbol{R}_+^{(m+1)n}$ is an early clearing payment vector if

$$q_i^0 = 1_{\{(1-\delta_i)a_i(\mathbf{q}) \geq d_i^0\}} \left(a_i(\mathbf{q}) - d_i^0\right)_+ \tag{1}$$

and

$$\begin{aligned} q_i^k = {} & 1_{\{(1-\delta_i)a_i(\mathbf{q}) \geq d_i^0\}} b_1^k \\ & + 1_{\{(1-\delta_i)a_i(\mathbf{q}) < d_i^0\}} \left(b_i^k \wedge \left((1-\delta_i)a_i(\mathbf{q}) - d_i^k \right)_+ \right) \end{aligned} \tag{2}$$

for $k = 1, \ldots, m$ and $i = 1, \ldots, n$.

Definition of early clearing payment vector (2)

In matrix form, $\mathbf{q}^0$ and $\mathbf{q}_i^k$ are expressed as

$$\mathbf{q}^0 = \mathbf{S}(\mathbf{q}) \left(\mathbf{a}(\mathbf{q}) - \mathbf{d}^0\right)_+ \tag{3}$$

and

$$\mathbf{q}^k = \mathbf{S}(\mathbf{q})\mathbf{b}^k + (\mathbf{I} - \mathbf{S}(\mathbf{q})) \left(\mathbf{b}^k \wedge \left((\mathbf{I} - \mathbf{\Delta})\mathbf{a}(\mathbf{q}) - \mathbf{d}^k\right)_+\right) \tag{4}$$

for $k = 1, \ldots, m$, where

$$\mathbf{S}(\mathbf{q}) = \begin{pmatrix} 1_{\{(1-\delta_1)a_1(\mathbf{q}) \geq d_1^0\}} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1_{\{(1-\delta_n)a_n(\mathbf{q}) \geq d_n^0\}} \end{pmatrix}. \tag{5}$$

Existence of early least clearing vector

Proposition

There exists an early clearing vector.

Proof (sketch): Let the function $\mathbf{g} : \boldsymbol{R}_+^{(m+1)n} \to \boldsymbol{R}_+^{(m+1)n}$ be given by

$$\mathbf{g}^0(\mathbf{q}) = \mathbf{S}(\mathbf{q}) \left(\mathbf{a}(\mathbf{q}) - \mathbf{d}^0\right)_+ \tag{6}$$

and

$$\mathbf{g}^k(\mathbf{q}) = \mathbf{S}(\mathbf{q})\mathbf{b}^k + (\mathbf{I} - \mathbf{S}(\mathbf{q})) \left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{q}) - \mathbf{d}^k\right)_+\right). \tag{7}$$

Then an early clearing vector is expressed as a fixed point $\mathbf{g}(\mathbf{q}) = \mathbf{q}$.

Algorithm

Proposition

The limit $\underline{\mathbf{q}}$ *is the least early clearing vector in the sense that*

$$\underline{\mathbf{q}} \leq \mathbf{q} \tag{8}$$

for any $\mathbf{q}$ *with* $\mathbf{g}(\mathbf{q}) = \mathbf{q}$ *where sequence* $\{\underline{\mathbf{q}}_h\}$ *is defined by* $\underline{\mathbf{q}}_0 = \mathbf{0}$ *and* $\underline{\mathbf{q}}_h = \mathbf{g}(\underline{\mathbf{q}}_{h-1})$.

Illustration of Early Clearing where $n = 2, m = 1, \Pi^0 = O$

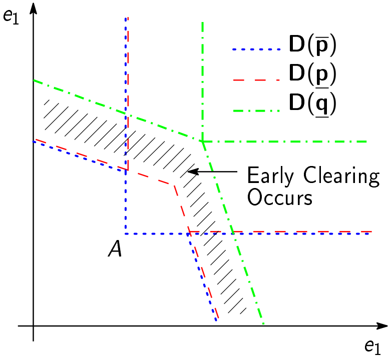
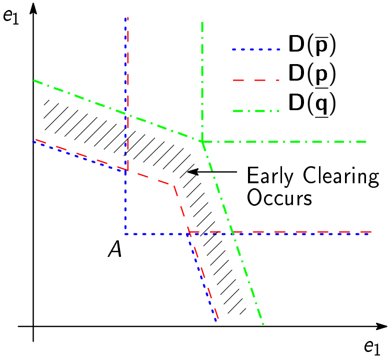


Illustration of Early Clearing where $n = 2, m = 1, \Pi^0 = O$



Assume that

- the risk-free rate is a constant r.
- business assets $\mathbf{e}$ are lognormally distributed.

Monte Carlo simulation for $\overline{\mathbf{p}}$

1. Distibute $\mathbf{e}$
2. For each path,
   1. set $\overline{\mathbf{p}}_0 = (\mathbf{x}, \mathbf{b}^1, \ldots, \mathbf{b}^m)^\top$
   2. iterate $\overline{\mathbf{p}}_h = \mathbf{f}(\overline{\mathbf{p}}_{h-1})$ until $|\mathbf{f}(\mathbf{p}) - \mathbf{p}| < \epsilon$
3. Take $E[\mathrm{e}^{-rT}\mathbf{p}]$.

Numerical Results

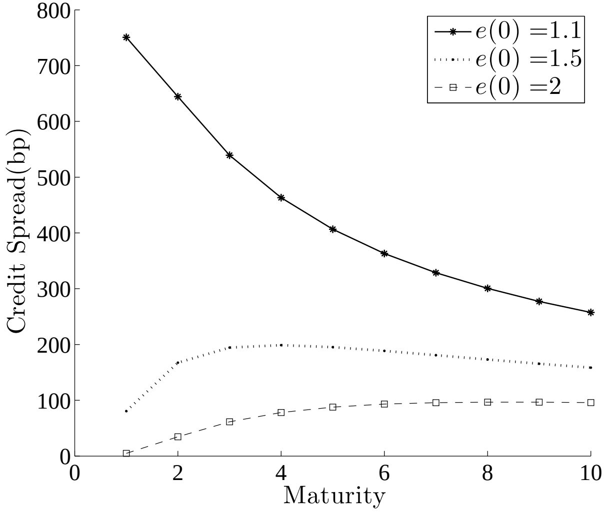
Basic parameters

- Parameters:
  $n = 10, m = 1, \sigma = 0.3, r = 0.05, \delta = 0.4, b_i = 1.0, T = 10$
  $\rho_{ij} = 0.0, \pi^1_{ij} = 0.2, \pi^1_{ii} = 0, \pi^0_{ij} = 0, i, j = 1, \ldots, n$
- Sample paths $N = 1,000,000$

Notes:

- We suppose $\mathbf{\Pi}^0 = \boldsymbol{O}$. So $\overline{\mathbf{q}} = \underline{\mathbf{q}}$ when $\delta \geq 0$.
- We call $\underline{\mathbf{q}}$ early clearing vector.

Credit Spreads by Greatest Clearing Vector under Cross-holding of Debts



Credit Spreads (bp) with Greatest, Least and Early Clearing Vector

	payoffs	1(y)	3	5	7
$e(0) = 2.0$	G^*	4.75599	61.33521	87.76826	95.71702
	L^*	4.76005	61.3808	87.8357	95.78521
	E^*	181.55412	207.03332	185.93967	166.3084
$e(0) = 1.5$	G	80.47351	194.53677	195.15374	180.82115
	L	80.61174	194.84206	195.37659	180.99323
	E	874.91084	474.94717	347.87775	280.36428
$e(0) = 1.1$	G	750.73819	539.26339	406.62969	328.6363
	L	755.51422	540.5624	407.2223	329.00211
	E	2728.66617	971.97699	615.01895	456.95952

G: paid by greatest clearing vector, L: paid by least clearing vector, E: paid by early clearing vector,

The Finding

Early clearing vector can give serious damage to the economy.

Comparison with Merton model (The idea)

- We basically follow Karl and Fischer (2013) and compare our model to Merton (1974) (without cross-holdings).
- Idea
  - Set the initial assets of the two models to be the same.
  - Choose the volatilities and covariance of e_i in Merton's model to match a_i in our model.

Simulation Methods

Sequence:

1. Simulate cross-holding model with $\delta = 0$ under basic parameter set.
   - Calculate firm value $x_i = p_i^0 + p_i^1$ and $E[x_i], \Sigma(x_i, x_j)$ for the input values for Merton model.
2. Simulate Merton model under $E[x_i], \Sigma(x_i, x_j)$.
3. Simulate cross-holding model with $\delta > 0$ under basic parameter set.
4. Simulate Merton model under $E[x_i], \Sigma(x_i, x_j)$ with $\delta > 0$

VaR and CS of firm 1's Debt

Base Case ($\delta = 0$)

	$\rho = 0,\ \delta = 0$				$\rho = 0.5,\ \delta = 0$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π^1_{ij}	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0381	-0.0747	129.69	118.79	-0.0767	-0.0879	145.39	139.95
0.50	-0.0030	-0.0441	71.38	74.21	-0.0480	-0.0578	91.47	93.87
0.70	0.0244	-0.0165	23.00	40.01	-0.0108	-0.0248	36.41	49.50

Greatest Clearing Vector

	$\rho = 0.0,\ \delta = 0.4$				$\rho = 0.5,\ \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π^1_{ij}	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0867	-0.1154	280.85	222.74	-0.1389	-0.1279	299.08	251.61
0.50	-0.0579	-0.0868	222.87	157.82	-0.1291	-0.0995	256.52	187.68
0.70	-0.0384	-0.0616	140.39	99.80	-0.1243	-0.0691	194.65	117.11

Merton model may overestimate credit risk with $\delta = 0$

Merton model underestimate credit risk with $\delta > 0$ and $\rho > 0$

VaR and CS of firm 1's Debt

Least Clearing Vector

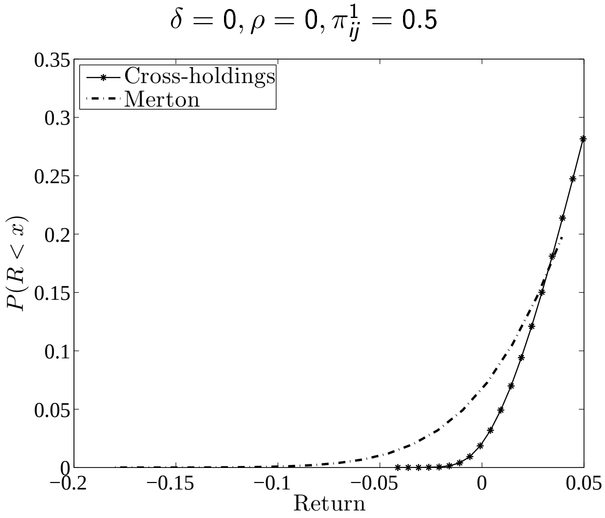
	$\rho = 0.0,\ \delta = 0.4$				$\rho = 0.5,\ \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π^1_{ij}	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0866	-0.1154	281.52	222.74	-0.1388	-0.1279	299.84	251.61
0.50	-0.0579	-0.0868	225.76	157.82	-0.1288	-0.0995	259.87	187.68
0.70	-0.0389	-0.0616	150.47	99.80	-0.1230	-0.0691	207.86	117.11

Early Clearing Vector

	$\rho = 0.0,\ \delta = 0.4$				$\rho = 0.5,\ \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp	
π^1_{ij}	XOD	Merton*	XOD	Merton*	XOD	Merton*	XOD	Merton
0.30	-0.0832	-0.1077	370.70	300.03	-0.1314	-0.1203	386.68	326.79
0.50	-0.0542	-0.0786	338.46	240.04	-0.1188	-0.0914	366.45	268.16
0.70	-0.0341	-0.0531	301.47	184.60	-0.1097	-0.0606	342.80	201.56

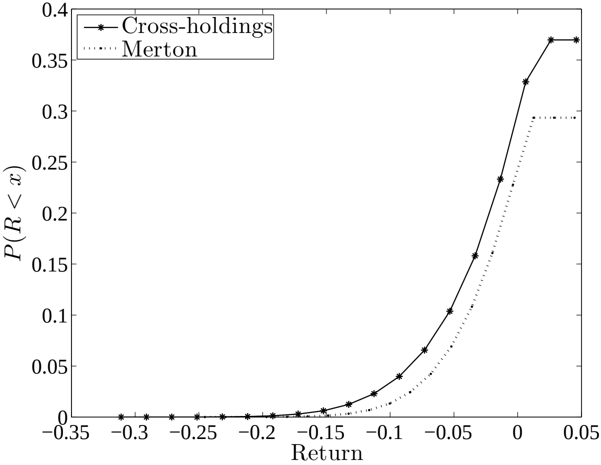
Merton model also underestimate credit risk with $\delta > 0, \rho > 0$, especially in the early clearing

Distribution Functions of Firm 1's Debt Return (Base Case)



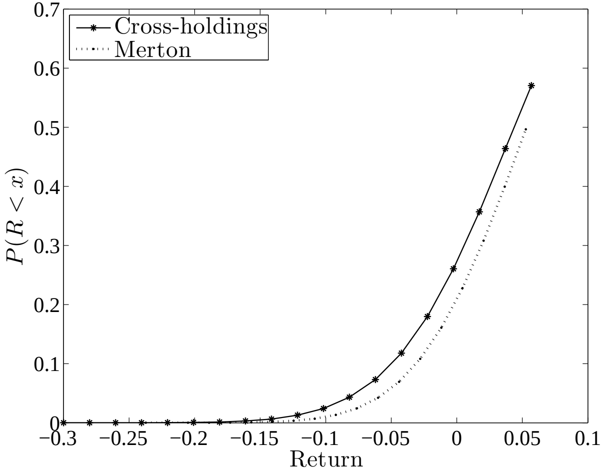
Distribution Functions of Firm 1's Debt Return by Greatest Clearing

$$\delta = 0.4, \rho = 0.5, \pi_{ij}^1 = 0.5$$

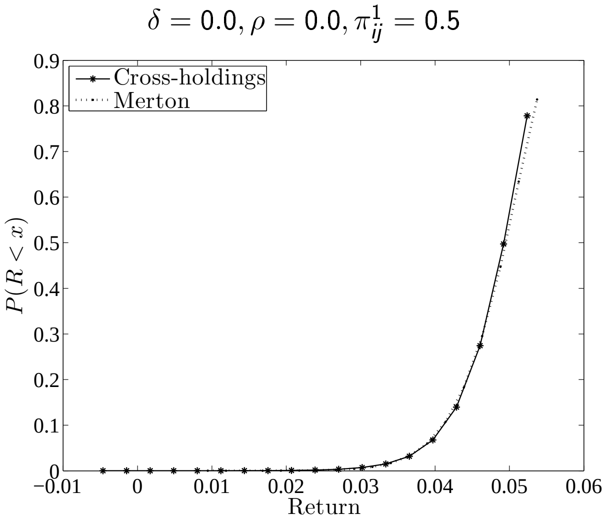


Distribution Functions of Firm 1's Debt Return by Early Clearing

$$\delta = 0.4, \rho = 0.5, \pi_{ij}^1 = 0.5$$

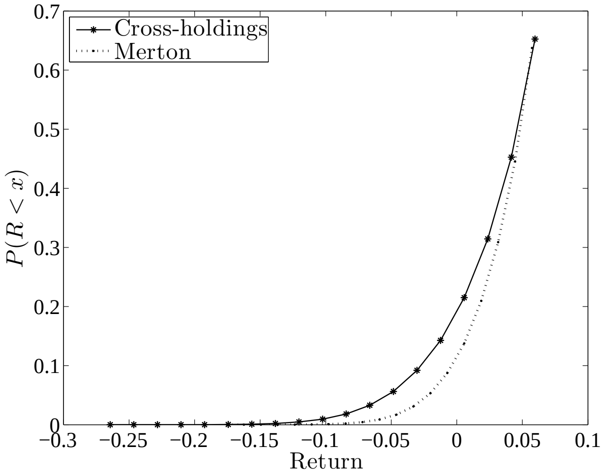


Distribution Function of Bonds Portfolio ($n = 10$) Return (Base Case)



Distribution Function of Bonds Portfolio ($n = 10$) Return by Early Clearing

$$\delta = 0.4, \rho = 0.5, \pi_{ij}^1 = 0.5$$



Conclusion

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In this talk, we

- present the pricing model of the corporate securities with cross-holdings , default costs and bond seniorities
- propose an early clearing payment vector $\underline{\mathbf{q}}$ to capture the financial crisis.
- show the existence of $\underline{\mathbf{q}}$ and the method to derive it.

By numerical example, we

- show $\underline{\mathbf{q}}$ can have serious damage to the economy.
- show Merton model tends to overestimate the credit risk with $\delta = 0$.
- show Merton model can underestimate the credit risk with $\delta > 0, \rho > 0$.

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— — — Thank you for your attention — — —